

Bounded confidence model on a still growing scale-free network

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Abstract. A Bounded Confidence (BC) model of socio-physics, in which the agents have continuous opinions and can influence each other only if the distance between their opinions is below a threshold, is simulated on a still growing scale-free network considering several different strategies: for each new node (or vertex), that is added to the network all individuals of the network have their opinions updated following a BC model recipe. The results obtained are compared with the original model, with numerical simulations on different graph structures and also when it is considered on the usual fixed BA network. In particular, the comparison with the latter leads us to conclude that it does not matter much whether the network is still growing or is fixed during the opinion dynamics.

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1 Introduction

It has recently been found that many systems, ranging from social science to biology, from economics to technology, which can be described as complex networks, seem to share some important topological features such as the scale-free degree distribution, where the probability that a node of these networks has k connections follows a power-law $P(k) \sim k^{-\gamma}$, with γ laying in a quite wide interval $2 \leq \gamma \leq 3$ [1]. For example, the Internet [2], in which nodes are computers and routers and edges are physical or wireless connections between them; the World Wide Web [3], in which the nodes are the web pages (documents) and the edges are the hyperlinks that point from one document to another; the scientific collaboration network [5], in which nodes are scientists and edges represent collaboration in scientific papers (two scientists are linked if and only if they are co-authors of the same paper).

Of particular interest here are the social networks, where the nodes are people, and the ties between them are (variously) acquaintance, friendship, political alliance or professional collaboration. More specifically, in this paper we study a model of opinion dynamics evolved on a scale-free network. Many models about opinion dynamics have been proposed [6, 7, 8, 12, 13, 14, 15, 16, 10, 17], however, in general only binary opinions were considered [12, 13, 14, 15, 16], such way we have only minority or majority opinions and it is impossible to distinguish between moderate and extreme opinion. As simplistic as it appears, the binary decision framework has been used to address surprisingly complex problems [13]. An interesting and straightforward extension would be to consider continuous opinions, i.e., a

wide spectrum of opinions [7, 8, 10, 11]. Modeling of such a model was earlier started by applied mathematicians and focused on the conditions under which a panel of experts would reach a consensus [8, 12, 18].

In the present work, we simulate on the Barabási-Albert network a simple consensus-finding model, the “Bounded Confidence” (BC) model [7], where originally the individuals (sites or nodes) living in the continuum, in contrast to our network, have continuous opinions and the individuals can influence each other only if the distance between their opinions is below a threshold.

In the next section, we describe the standard Bounded Confidence model and the Barabási-Albert network, in section 3, we present our results and in section 4, our conclusions.

2 The Model

The Barabási-Albert (BA) network [19] starts to grow from an initial cluster of m fully connected sites. Each new node (or vertex), that is added to the network creates m links (edges, ties) that connect it to previously added nodes. The power-law distribution emerges as a result of preferential attachment, which means that the probability of a new link to end up in a vertex i is proportional to the connectivity k_i of this vertex. The validity of the preferential attachment was confirmed within real networks analysis [19]. The BA algorithm generates networks with the desirable scale-free distribution $P(k) \propto k^{-3}$ and small values of the average shortest path. The only striking discrepancy between the BA model and real networks is that the value of the clustering coefficient - which is the probability that two nearest neighbors of the same node are

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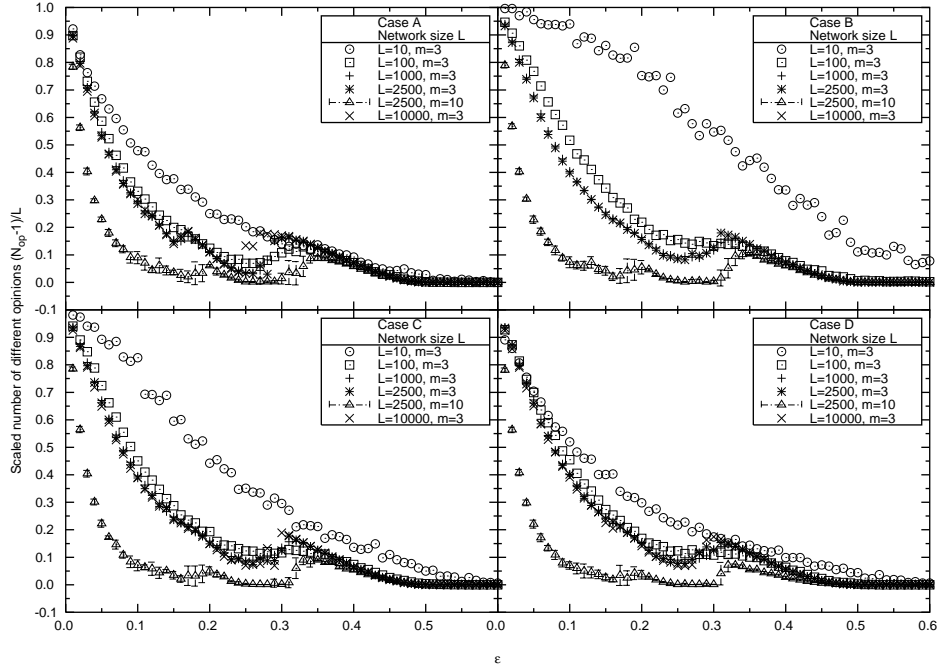


Fig. 1. Scaled total number of different opinions versus the inverse of the constant confidence bound ϵ for the **Case A** (top left), **Case B** (top right), **Case C** (bottom left) and **Case D** (bottom right) using different network sizes.

also mutual neighbors - predicted by the theoretical model decays very fast with network size and for large systems is typically several orders of magnitude lower than found empirically.

In the Bounded Confidence model [7,8,17] each individual (or node) is represented by a continuous opinion s_i , whose initial value is a random number chosen between zero and one ($0 < s_i < 1$). At every time step two randomly chosen individuals i and j have their opinions readjusted only if their difference in opinion, $\delta_{ij} = (s_i - s_j)$, is smaller in magnitude than a threshold ϵ : $|\delta_{ij}| < \epsilon$. In this way, the opinions are adjusted according to:

$$s_i = s_i - \mu\delta_{ij} \quad \text{and} \quad s_j = s_j + \mu\delta_{ij} \quad (1)$$

where μ is the convergence parameter whose values may range from 0 to 0.5 and characterizes the flexibility in changing the opinion. It has been observed that the qualitative dynamics mostly depend on the threshold ϵ , which controls the number of peaks in the final distribution of opinions. The number L of individuals and the parameter μ only influence convergence time and the width of the distribution of final opinions [7].

At each time step $0 < t \leq L - m$, we have the following process:

1. The BA network grows, i.e., one new site i (individual) is added, and a random opinion s_i ($0 < s_i < 1$) is set as initial opinion to the single new node i of the network.
2. For each new site added to the network, $N_d = 10^6$ BC runs are performed. For each run, all the nodes are randomly visited and updated (a random list of nodes assures that each node is reached exactly once) by selecting a node i at random and, among its connected

nodes, a node j at random. If $|\delta_{ij}| < \epsilon$, their opinions, s_i and s_j , are re-adjusted (Eq. 1).

In contrast to a recent work simulating the BC model on the usual fixed BA network [20,22], the consensus process of the BC model is not performed after the complete network had been constructed, but while the network grows, i.e., while each new node is added to the network, a Bounded Confidence prescription is applied: the already existing sites have their opinions readjusted every time when a new site is added. Notice that this assumption has already been used before, however in the context of a binary opinion model on a Barabási-Albert (BA) network [16] and on a deterministic pseudo-fractal network [15].

The following four cases have been investigated:

- **Case A:** For each selected site i , **all** the nodes connected to it are randomly visited and tested.
- **Case B:** For each selected site i , **only one** neighbor j is selected from the m sites which i had selected to make a link when it was added to the network. If $|\delta_{ij}| > \epsilon$, another i is picked up.
- **Case C:** For each selected site i , **only one** neighbor j is taken from **all** its k_i neighbors. If $|\delta_{ij}| > \epsilon$, another i is picked up.
- **Case D:** For each selected site i , **only one** neighbor j is taken from **all** its k_i neighbors. In case of $|\delta_{ij}| > \epsilon$, another neighbor j is randomly picked up. If after k_i times no neighbor j provides $|\delta_{ij}| < \epsilon$, then another i is selected.

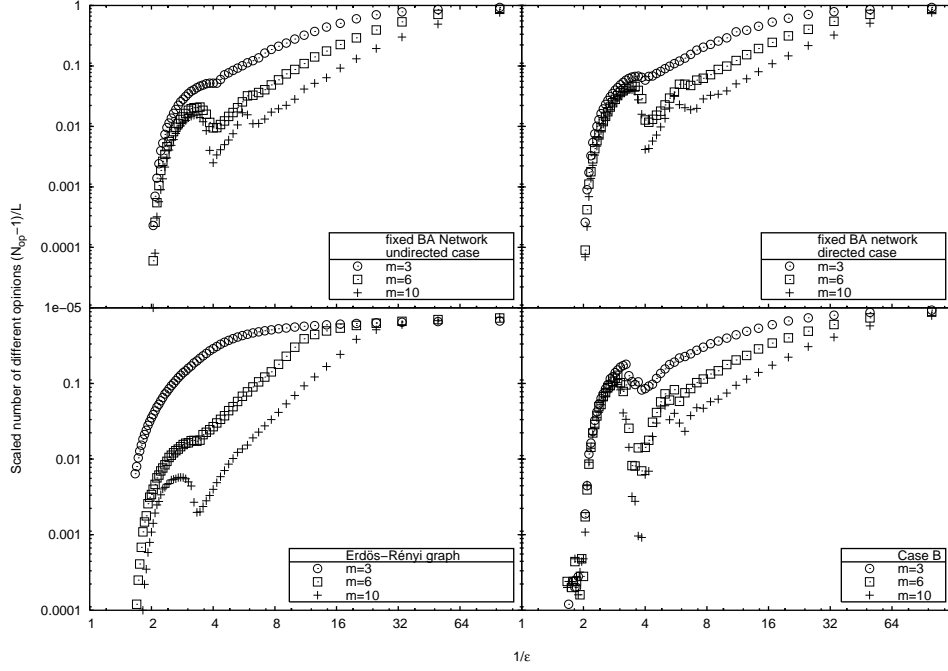


Fig. 2. Scaled total number of different opinions versus the inverse of the constant confidence bound ϵ for the **undirected case** (top left), **directed case** (top right), **Erdős-Rényi** (bottom left) and **Case B** (bottom right) using different values of m . Both axes are logarithmic.

3 Results

After t time-steps the network has $L = m + t$ nodes (individuals). The curves presented here correspond to the results averaged over 100 samples. The opinions are placed in bins of width 10^{-6} and are counted by checking which bins are occupied and do not have the lower neighboring bin occupied. In this way, the total number of fixed opinions is obtained.

In Figure 1: case A (top left), case B (top right), case C (bottom left) and case D (bottom right), we present the total number of different fixed opinions $N_{op} - 1$ divided by the network size L as a function of the inverse of the constant confidence bound ϵ . It has been observed in all cases (Fig. 1) that when $\epsilon > 0.5$ a full consensus (only one opinion survives) is reached and for $\epsilon < 0.5$, no consensus is reached and the number of different fixed opinions increases with decreasing ϵ . In fact, when ϵ goes to zero, the number of surviving opinions approaches the network size L , as expected [20]. The threshold value of ϵ obtained in our simulations ($\epsilon > 0.5$, a complete consensus and $\epsilon < 0.5$, no consensus) is in a complete concordance with the original BC model [7,9] and with recent numerical simulations of it on different graph structures [10] that provide strong numerical evidence that this value does not depend on the way the agents are connected to each other (i.e, the graph structure), but it relies on the social dynamics. The small statistical error of the threshold ϵ for complete consensus in our simulations (shown only for $L = 2500$ and $m = 10$ in Fig.1) confirms the good agreement of our results with the previous ones [7,9,10].

We can also observe that when the network size L increases, the total number of different fixed opinions $N_{op} - 1$ increases $\propto L$, while the scaled number of different opinions - $(N_{op} - 1)/L$ - for smaller system sizes L ($L \leq 100$) presents stronger finite size effects, however, they become weaker for larger system sizes L , then the L -dependence of the scaled excess number almost disappear.

Moreover, for large L , interestingly we have observed in all cases that the number of final opinions reaches a local

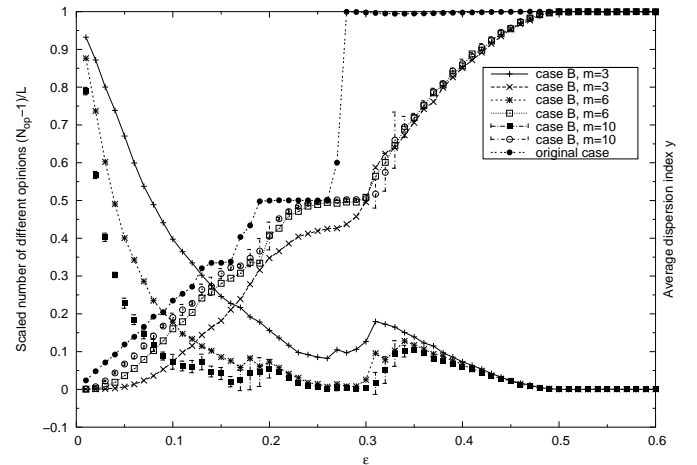


Fig. 3. Scaled total number of different opinions and the average dispersion index y as a function of the constant confidence bound ϵ . $L = 2500$ (Case B) and $L = 10000$ (Original model). Errorbars are shown for $m = 10$.

minimum around $\epsilon = 0.25$ (a steeper one) and $\epsilon \approx 0.15$ (a smoother one). Besides, as the connectivity "m" increases the minimum becomes deeper and others minimums appear (see all cases in Fig.1 when $L = 2500$ and $m = 10$). These local minima could not be noticed before in the usual fixed BA network studied in Ref.[20], due to the lack of detailed simulations for different values of the threshold ϵ . In this way, in order to analyze more carefully these minima we have also performed simulations of the BC model on the usual fixed BA network, on the Erdős-Rényi random graph, as well as without any network topology, i.e., the original BC model. In the former one, as it was previously studied in [20], two different cases have been investigated: the directed case (our case B) and the undirected one (our case C).

The construction of the Erdős-Rényi random graph [21] starts with a set of L isolated vertices, then successive edges are randomly added with a probability p . In this way, the total number of edges is $m_t = pL(L-1)/2$ and the average number of neighbors of a node (degree or connectivity) is $m = p(L-1)$. In the limit $L \rightarrow \infty$, the mean number of bonds per site can be approximated by pL and a Poissonian connectivity distribution is observed. In our simulations, the graph has been built in such way that each node has at least m links. Moreover, on this topology we have investigated only the case C.

Figure 2 shows the scaled total number of different opinions versus the inverse of the threshold ϵ for the usual fixed BA network [20]: undirected case (top left) and directed case (top right), for the Erdős-Rényi (bottom left) and for case B on a still growing BA network (bottom right) when different values of m are considered. As we can see, the large is the average connectivity, which is made by increasing m , the steeper are the local minima observed when $\epsilon \approx 0.25$ and $\epsilon \approx 0.15$ (it does not seem to appear for the Erdős-Rényi graph). An individual (node) with few connections (link) has less chances to interact with a neighbor whose opinion is close enough ($s_i - s_j < \epsilon$) to its own opinion to actually interact, in this way, many of the individuals remain outside the distribution of clustered opinions. This implies that if the average connectivity increases more individuals should converge into the same opinion's cluster, which make us to expect that in the limit of large m (everybody interacts with everybody) the results become closer to those of the original BC model (without any network topology): the majority of the individuals has the same opinion or only one cluster is observed for $\epsilon > 0.25$ [22].

In the original BC model, it has been found that for large L the number of clusters varies as the integer part of $1/2\epsilon$ [7]. In such way that for $\epsilon > 0.25$ most of individuals belong to only one single cluster, and for $\epsilon < 0.25$ several large ones (see original case in Fig. 3) [22]. On the other hand, however simulations on the BA network show the results follow this $1/2\epsilon$ rule, the existence of many individuals with lower connectivity in scale-free networks makes the fraction of individuals into the same cluster to become smaller [22]. Nevertheless, it is important to emphasize that since the opinion are represented by real numbers, the

convergence towards to a full consensus (only one opinion) inside a cluster is never actually reached, i.e., the clusters correspond to a group of individuals with very similar but not exactly equal opinions. The differences between the clusters are related to the threshold ϵ .

A simple way to check clustering and specially its average, is the dispersion index y proposed by Derrida and Flyvberg [23]:

$$y = \frac{\sum_i s_i^2}{(\sum_i s_i)^2} \quad (2)$$

where s_i is the cluster size, i.e., the number of individuals in each cluster i and L the total number of individuals. In Figure 3, we plot the scaled final number of opinions and the dispersion index y against the threshold ϵ for different values of the connectivity m . When $0.10 < \epsilon < 0.15$, $0.15 < \epsilon < 0.2$ and $0.25 < \epsilon < 0.3$ local minima can be observed, and $\epsilon > 0.5$ a full consensus is reached. As we can notice, the regions corresponding to these minima are the same ones where distinct behaviors in the dispersion index y are observed, which seem to be related to the transition of the number of clusters: four to three large clusters, three to two, two to one, respectively. In the original case (filled circles in Fig. 3) the transition regions are more sharper than in those ones observed for the case B, in which the dispersion index y varies more smoother and is indicated by a slope becoming the steeper the larger the connectivity m is. These results are in a good agreement with previous ones for the original BC model and for the BC model on the usual BA network [22]. The index y varies smoothly as a function of the threshold ϵ due mainly to the existence of many individuals with lower connectivity m that remain outside of the opinion convergence process and do not cluster. In this way, in an infinite network and in the limit of large mean connectivity m , one would get a sharp step function in these transition regions for the mean dispersion index y versus the threshold ϵ , i.e., the results for the scale-free network becomes very similar to those ones obtained for the original BC model, where everybody interacts with everybody [22]. Moreover, the continuous increase of the index y as a function of the threshold ϵ when $\epsilon > 0.3$, while for the original case y is equal to unity (corresponding a full consensus of the system, i.e, all the individuals belong to the same opinion cluster), shows clearly the existence of many individuals kept out of the clustering process [22] in the BA scale-free network.

4 Conclusions

Using a consensus model with bounded confidence on a still growing Barabási-Albert network, we have shown that the system reaches a full consensus when $\epsilon > 0.5$ and for $\epsilon < 0.5$, no consensus is reached anymore and the number of different fixed opinions increases with decreasing ϵ . This critical value for finding a complete consensus is the same one obtained in the original random case when the individuals were considered to live in the continuum [7,9,22], as well as when the BC model was simulated on

the usual fixed BA network [22] and for numerical simulations of the BC model on several graph structures [10]. Once the BC prescription here is performed while the network grows, so when a new node (individual) is added, it can find the existing sites of the network already in a complete consensus. Thus, we believe that this particular feature seems to be responsible for reducing strongly the finite size effects related to the L -dependence observed in [20]. We have also found local minima in the scaled final number of opinions as a function of the threshold ϵ , which are related to the phase transition in the number of opinion clusters. To investigate this question further, it has been performed additionally simulations of the original BC model, as well as considering it on the usual BA network (directed and undirected case) and on the Erdős-Rényi random graph for several values of the confidence bound parameter ϵ . All studied cases independently of the underlying topology show these local minima, which occur in the phase transition regions of the number of the opinions clusters as a function of the threshold ϵ . In particular for the small-world graphs, one observes that the minima become steeper and a sharper step function for the dispersion index y , the larger mean connectivity is, and also, in the limit of large connectivity and large network size one would get the same results obtained for the original BC model. Moreover, the reason to have the highest values of the dispersion index y smaller than unity (as obtained in the original BC model) is due to the fact that only a fraction of all individuals belongs to the big opinion cluster(s) resulting from the convergence process [7, 22]. In summary, our results lead us to conclude that it does not matter much whether the network is still growing or is fixed during the opinion dynamics, the opinion spreading properties remain the same. An identical conclusion has also been found for computer simulations on binary opinion dynamics [15, 16].

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